Hindawi
Advances in High Energy Physics
Volume 2021, Article ID 9915801, 14 pages
https://doi.org/10.1155/2021/9915801

Research Article

Study of the Ground-State Energies of Some Nuclei Using Hybrid Model

R. Hussien,[1] **Sh. M. Sewailem**,[1] **and L. I. Abou-Salem**[2]

[1]*Mathematics Theoretical Physics Department, Atomic Energy Authority, Cairo, Egypt*
[2]*Physics Department, Faculty of Science, Benha University, Benha, Egypt*

Correspondence should be addressed to R. Hussien; rabab.hussien216@gmail.com

Received 21 March 2021; Revised 6 July 2021; Accepted 4 August 2021; Published 6 September 2021

Academic Editor: Shi Hai Dong

The quark-quark (QQ) interaction as a perturbed term to the nucleon-nucleon interaction (NN) without any coupling between them is studied in a hybrid model. This model is used to calculate the ground-state energies of $^{2}H_{1}$ and $^{4}He_{2}$ nuclei. In a semirelativistic framework, this model is encouraged for light nuclei and the instanton-induced interaction by using the QQ potential and the NN interaction for a small scale around the hadron boundaries. This hybrid model depends on two theories, the one-boson exchange potential (OBEP) and the Cornell-dressed potential (CDP) for QQ. A small effect of quark-quark interaction is obtained on the values of the ground-state energies, around 6.7 and 1.2 percentage for $^{2}H_{1}$ and $^{4}He_{2}$, respectively nuclei.

1. Introduction

One of the fundamental problems of the nuclear structure is the derivation of the ground-state energies through different methods, such as properties related to the constituents of matter, which are represented in the physics of elementary particles with their characteristics and how each particle interacts with others. The interaction between each nucleon with all other nucleons generates an average potential field where each nucleon moves. The rules of the Pauli exclusion principle govern the occupation of orbital quantum states in the shell model and postulate that under the meson exchange between two nucleons, the wave function is the antisymmetrical product wave function. The calculation of the nuclear mean-field potential with Dirac-Hartree-Fock qualifies the description of nucleon-nucleon interaction to be successful microscopically. The interaction between two nucleons has three regions with three ranges. The first region originated from pseudoscalar meson, the second region was related to the scalar meson, and the third region was caused by the exchange of vector meson besides the effects of quantum chromodynamics (QCD). The nucleon-nucleon potential has no definite method to determine it. The Bonn group potential known as one-boson exchange potential is supposed to be the suitable model for this interaction because of the reduction of free parameters and fitting them accurately with the experimental data.

On the other hand, the quark degrees of freedom are under the dynamics of QCD. The interaction between quarks has various forms of potentials, and these forms have to regard the quark properties (confinement and asymptotic properties). The mechanism of the one-gluon exchange approach is dominant at the short range with two parts. The linear confinement at a long distance and a part of the asymptotic property represented in the pairing force acting only on the quark-antiquark states. The constituents of baryons composed of u, d, s quarks can use a semirelativistic potential model that refers to their interaction, including the instanton-induced forces. The instanton-induced model is used to describe baryons composed of light quarks that are demanded in the considered baryons. This interaction resembles the tunneling phenomena as it can be affected outside the hadron for a short scale comparing with the confinement scale. In the used model, we have two contributions in the potential form, the one-gluon exchange part and the exchange of pseudo particles between quark-antiquark pair. The possibility of proposing a hybrid model with no coupling between quarks inside the baryon and mesons outside it can be founded based on the variational concept of physics. In the present work, the ground-state energies for some light nuclei can be calculated successfully by using the considered hybrid model.

In Section 2, we introduce the theory of NN interaction through the one-boson exchange potential with investigations and motivations of the formula. In Section 3, we have a brief look at the QQ interaction, and the reason for choosing Cornell-dressed potential is mentioned to make the idea of the hybrid model possible. In Section 4, where the theoretical analysis for the construction of one-boson exchange potential through the exchange of two, three, and four mesons is clarified. Section 5 shows the theoretical analysis of QQ interaction and the final form of CDP. Finally, in Sections 6 and 7, the obtained results and conclusion are given.

2. Theory of NN through the OBEP

The start of using the fact that there is no unique potential for determination of the effective NN potential leads to exist different forms with different methods. So, this work concerned to show the effect of our potential which is published in previous work [1]. Our potential is constructed with the idea of one-boson exchange and also depends on the motion of nucleons in the nucleus. This motion produced a field called nuclear mean field, and the interaction between nucleons is controlled with Pauli principle and nuclear shell model. We considered the spatial exchange between two nucleons, so the nonlocal field is determined with the Hartree-Fock approximation, since the Fock effect is demonstrated in the nonvanishing spatial components for the vector part of the potential. Our potential implies the Dirac-Hartree-Fock method to determine the wave function and energy of a quantum N-body system in a stationary state. We classified our potential as a semirelativistic model because of neglecting the fourth power of momentum to simplify the formula, and it will be included in following work. Our potential is associated with the Bonn group to have the meson's function and its parameters. To calculate the ground-state energies of the Hartree-Fock approximation, we need to minimize the total energy of single particle potential by the Steepest descent methods directly to have the lowest energy. It is demanded for a modification of nucleon wave functions and energy.

The modification of wave function is demonstrated by Clebsch-Gordan coefficients and Talmi-Moshinsky harmonic oscillator bracket, affecting on the radial, spin, and isotopic wave functions. We use the formalism of second quantization just as a convenient way of handling antisymmetric wave function. This formalism referred as a representation of the occupation number; hence, it leads to be represented in the Fock-state basis which can be constructed by filling up each single-particle state with a certain number of identical particles. As a real-space basis, we write the antisymmetric wave functions in a Slater determinant. Second quantization gives us the ability to displace the wave function as a Dirac state and do the same for Slater determinant. So, we use operators to specify the occupied orbitals and the field operators to define the coordinates for the real-space representation. It is noticed that the atom in a quantum state of energy (E) depends only on that energy through the Boltzmann factor and not on any other property of the state when we represent this atom in complete thermal equilibrium to determine the ground-state energy for the considered nuclei. We use this fact to neglect the tensor force for the deuteron nucleus and calculate its wave function in S-state only.

The fact of being the vector mesons and QCD affected on the nuclear properties at short range; hence, the studying of nucleon-nucleon interaction through OBEP should not be enough. The exchange of bosons with OBE potential models comes about more than size of nucleon or equal to the internucleon distances. We have the effect of QCD at distance less than or almost around the boundaries of hadron, and that is necessary for the description of nucleon-nucleon interaction. The quantitative theoretical models can analyze the experimental data based on the degrees of hadrons over the last three decades [2–8], and also, the quark degrees of freedom in QCD models are successful models for the description of the nuclear properties [9–12]. These models analyze the static properties of baryon successfully. We are concerned to add the quark degrees of freedom as a perturbed term to the meson degrees of freedom and have a Hamiltonian equation of two parts as the following:

$$H = H_{\mathrm{NN}} + H_{\mathrm{QQ}}, \tag{1}$$

where the Hamiltonian of the nucleon-nucleon interaction is H_{NN} and the hamiltonian of the quark-quark interaction is H_{QQ}. So, we study the OBEP with the exchange of three and four mesons as it represents the nucleon-nucleon interaction and the Cornell-dressed potential as the quark-quark interaction for constructing more realistic model of the nuclear properties.

3. Theory of QQ through the CDP

The simulation of quark-quark interaction phenomenon in a semirelativistic framework shows that the long range part of this interaction is increasing linearly with the distance and is called confinement, and the short range of interaction is a result of Coulomb-like interaction (one-gluon exchange). The idea of considering the contribution of the constant potential is dominant than the other contributions to the quark-quark potential worth good thinking of it as in [13, 14], and a good spectra of mesons and baryons are obtained. At first, very good results for the charmonium spectrum obtained from a simple form of a potential are called Funnel potential or Cornell potential [15, 16]. The Hamiltonian of hadrons containing light quark should simultaneously define a number of relativistic corrections. The momentum-dependent corrections as well as a nonlocal kinetic energy (there is no commutation with faster than light, and it is compatible with special relativity) is required to be included in the effects of relativistic kinematics of the potential of the potential energy operator [17]. These relativistic kinematics are included in the Bethe-Salpeter equation, neglecting the spin effect which introduces nonlocal modifications of the relative coordinate. The spinless Salpeter equation has the form

$$H = \sum_{i=1}^{3} \sqrt{\vec{p}_i^{\,2} + m_i^2} + \sum_{i<j=1}^{3} V_{ij}. \quad (2)$$

This is suitable for baryon where H is the total energy of the system, V is the central potential between two particles (i, j), and $\vec{p}$ is their relative momentum. In the case of baryon, m_i is the constituent mass of quarks with the same mass of u and d quarks (isospin symmetry is maintained). The central potential is the Cornell potential,

$$V_C(r) = \frac{1}{2}\left[\frac{-k}{r} + \mathrm{ar} + C\right]. \quad (3)$$

The factor half is related to the half rule, k is the Coulomb parameter, a is the string constant, and C is additive constant equals zero in the heavy quark sector. To solve the spinless Salpeter equation, [17] expanded the wave function in terms of a complete set of basis functions according to Rayleigh, Ritz, and Galerkin methods as the previous radial wave function in OBEP. Many symmetries are slightly broken in nature as it can give rise to the classical solutions to a particular symmetry-breaking amplitude. This amplitude is similar to the tunneling effect; indeed, the classical solutions of the equation of motion can sometimes describe the tunneling through a barrier. The classical solution of equations of motion was introduced in Yang-Mills theory, known as "instanton" term. The computation of the quantum effects of instantons was introduced by 't Hooft [18] firstly. In [13, 14], the authors went with instanton-induced interaction (it is a solution to the equations of motion of the classical field theory, and it is supposed to be critical points of the action for such quantum theories). In nonrelativistic quark model, it is assumed that this model is based on the confinement potential and a residual interaction. The residual interaction is related to the reduction of the one-gluon exchange OGE. One is able to compute the residual interaction by 't Hooft force from instanton effects [18, 19]. Ref [15] proposed a model of quark interaction with the replacement of the traditional OGE potential by a nonrelativistic limit of 't Hooft's interaction. The residual interaction is observed by 't Hooft as an expansion of the Euclidean action around the single instanton solutions under the assumption of zero mode in the fermion sector. This interaction has an effective Lagrangian with effective potential between two quarks. The instanton calculus can be summarized by four steps [20].

(i) The gluon fields cannot deform the instantons into classical solutions continuously

(ii) The perturbative gluon diagrams cannot cover the effective interaction between quarks which caused by instantons

(iii) The instanton calculus denotes as a nonperturbative method for the calculation of path integrals, which are represented in the fluctuations around the instanton and change the action. All of this is normally done in the Gaussian approximation

(iv) The instanton effects in QCD realized that instanton is similar to be described as 4-dimensional gas of pseudo particles; then, use the summation over the instanton gas

In the first analogy of super conductivity with the Bardeen-Cooper-Schrieffer theory [21], when the interaction between fermions (nucleons) and light quarks is attracted strongly at the short range, this interaction can rearrange the vacuum and the ground state affected by it which resembles the effect of super conductivity. Then, the short range interaction can bind these constituent light quarks into hadrons without confinement in order to make quantitative predictions for hadronic observable. It is clarified that instanton is represented as a tunneling event between vacua [22].

4. Theoretical Analysis of OBEP

The general form which describes the ground-state energy of the considered system is the following:

$$H|\Psi\rangle = E|\Psi\rangle, \quad (4)$$

where H is the Hamiltonian and E is the total energy of the system.

$$E = T + V, \quad (5)$$

defining T as kinetic energy and V as the potential energy. Hence, the Hamiltonian fermions are interacting via the potential V_{ij}. Thus, the accurate Hamiltonian interaction of the nuclear system can be described by Dirac to represent the number of fermions' interaction where this Hamiltonian is [23–26] as follows:

$$H = \sum_{i}^{A} c\vec{\alpha}_i.\vec{p}_i + (\beta_i - I)m_i c^2 + T_{ij} + \frac{1}{2}\sum_{i\neq j}^{A} V_{ij}, \quad (6)$$

where $\vec{\alpha}$ and β are 4×4 Dirac matrices, c is the speed of light, m_i is the nucleon mass, T_{ij} is the relative kinetic energy, and $\vec{p}$ is the momentum operator.

See Appendix A for the details of relative kinetic energy calculations to have the following equation [27–30]:

$$T_{ij} = \frac{2}{mA}\sum_{i<j} p_{ij}^2. \quad (7)$$

Substituting the last equation in Equation (6), thus the relativistic Hamiltonian operator for bound nucleons which interact strongly through the potential can be expressed as follows:

$$H = \sum_{i}^{A} C\vec{\alpha}_i.\vec{p}_i + (\beta_i - I)m_i c^2 + \frac{2}{mA}\sum_{i<j}^{A} p_{ij}^2 + \sum_{i<j}^{A} V_{ij}. \quad (8)$$

In the Hartree-Fock theory, we seek for the best state giving the lowest energy expectation value of this Hamiltonian to determine the ground-state energy of the considered nuclei. One is able to ensure the antisymmetry of the fermions' wave functions with the aid of Slater determinant introduced in 1929 and Hartree product to have the convenient form in calculating the ground-state energy as the following wave function which is suitable for fermions [24]:

$$\Psi(r) = \frac{1}{\sqrt{A!}} \det \Psi_i\left(\vec{r}_i\right), \quad (9)$$

where the wave function of all nucleons is $\Psi(r)$ and the wave function for i-nucleon is $\Psi_i(r)$. The wave function for nucleon i depends on the oscillator parameter as

$$\Psi_i\left(\vec{r}_i\right) = C_{i\alpha} F_\alpha\left(\vec{r}_i\right), \quad (10)$$

where $C_{i\alpha}$ is the oscillator constant and F_α is the wave function of two components.

$$F_\alpha = \left| \begin{matrix} \Phi_\alpha \\ \chi_\alpha \end{matrix} \right\rangle, \quad (11)$$

where the wave function for radial component is Φ_α and the spin component is χ_α. The principle of antisymmetry of the wave function was not completely explained by the Hartree method according to Slater and Fock independently. So the accurate picture in calculating the ground-state energy is the Hartree-Fock approximation.

$$\sum_{i\alpha\beta} h_i C_{i\alpha}^* C_{i\beta} \langle F_\alpha \mid F_\beta \rangle = \sum_{i\alpha\beta} C_{i\alpha}^* C_{i\beta} \left\langle F_\alpha(r) \middle| c\vec{\alpha}_i.\vec{p} + (\beta_i - I)m_i c^2 \middle| F_\beta \right\rangle + \sum_{i<j}\sum_{\alpha\gamma\beta\delta} C_{i\alpha}^* C_{i\beta} C_{j\gamma}^* C_{j\delta} \left\langle F_\alpha F_\gamma \middle| \left(\frac{2}{mA} P_{ij}^2 + V_{ij}\right) \middle| \widetilde{F_\beta F_\delta} \right\rangle, \quad (12)$$

where $C_{i\beta}$ is the occupation number or the oscillator number (for a system consisting of fermions or particles with half-integral spin, the occupation numbers may take only two values: 0 for empty states or 1 for filled states). The two components of the wave functions have the following relation between them [31, 32].

$$\chi = \left(1 - \frac{\varepsilon - \nu}{2Mc^2}\right) \frac{\vec{\sigma}.\vec{p}}{2mc} \phi. \quad (13)$$

Using the relation Equation (13), where ε is external energy which equals zero, here we deal with ground state and c^3 makes the value so small and can be neglected.

$$\chi \cong \frac{\vec{\sigma}.\vec{p}}{2mc} \phi, \quad (14)$$

$$\langle \Psi_i(r) | E | \Psi_i(r) \rangle = \left\langle \Psi_i(r) \middle| \widehat{H}_1 \middle| \Psi_i(r) \right\rangle + \left\langle \Psi_i(r) \middle| \widehat{H}_2 \middle| \Psi_i(r) \right\rangle. \quad (15)$$

Differentiate Equation (12) with respect to $C_{i\alpha}$, and the $\widetilde{F_\beta F_\delta}$ has a sign defines the exchange that happening between the two nucleons, hence substituting $\sum_j C_{j\gamma}^* C_{j\delta} = 1$ and $\langle F_\alpha \mid F_\beta \rangle = 1$.

$$\sum_{i\alpha\beta} C_{i\beta} \left[\langle F_\alpha | \widehat{H}_1 | F_\beta \rangle + \sum_{i<j} \left\langle F_\alpha F_\gamma \middle| \widehat{H}_2 \middle| \widetilde{F_\beta F_\delta} \right\rangle - h_i \right] = \sum_{i\alpha\beta} C_{i\beta} \left[\left\langle F_\alpha \middle| c\vec{\alpha}_i.\vec{p} + (\beta_i - 1)m_i c^2 \middle| F_\beta \right\rangle + \sum_{i<j} \left\langle F_\alpha F_\gamma \middle| \left(\frac{2}{m_i} P_{ij}^2 + V_{ij}\right) \middle| \widetilde{F_\beta F_\delta} \right\rangle - h_i \right] = 0. \quad (16)$$

Treating with the 1st part of Equation (16) gives us the coming formula.

$$H_1 = \sum_{i\alpha\beta} C_{i\beta} \left\langle F_\alpha \middle| c\vec{\alpha}_i.\vec{p} + (\beta_i - I)m_i c^2 \middle| F_\beta \right\rangle. \quad (17)$$

Taking into account Dirac matrices [33] with defining the wave functions in bracket as $|F_\alpha\rangle = \left| \begin{matrix} \phi_\alpha \\ \chi_\alpha \end{matrix} \right\rangle$ and $\langle F_\alpha | = \langle \phi_\alpha \, \chi_\alpha |$,

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad (18)$$

where unit matrix $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Substituting α, β in H_1 will have the following result:

$$\begin{aligned}
&\langle F_\alpha|H_1|F_\beta\rangle \\
&= \left\langle F_\alpha \middle| c\begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}.\vec{p} + \left[\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right] m_i c^2 \middle| F_\beta \right\rangle \\
&= \left\langle \phi_\alpha \middle| \frac{(\vec{\sigma}.\vec{p})(\vec{\sigma}.\vec{p})}{2m} \middle| \phi_\beta \right\rangle + \left\langle \phi_\alpha \middle| \frac{(\vec{\sigma}.\vec{p})(\vec{\sigma}.\vec{p})}{2m} \middle| \phi_\beta \right\rangle \\
&\quad - \left\langle \phi_\alpha \middle| \frac{(\vec{\sigma}.\vec{p})(\vec{\sigma}.\vec{p})}{m} \middle| \phi_\beta \right\rangle = 0.
\end{aligned} \tag{19}$$

The 1st term of kinetic energy tends to zero, and this result has an agreement with another calculations in [34] After the treatment of the kinetic terms are done, the residual Hamiltonian of the expectation value becomes

$$H = H_2 = \sum_{i<j} \left(\frac{2}{m_i} p_{ij}^2 + V_{ij} \right) \tag{20}$$

The popular form of the force between two nucleons is cleared according to meson exchanges. The potential form of one-boson exchange V_{ij} between two nucleons (i, j) is based on the degrees of freedom associated with three mesons, pseudoscalar, scalar, and vector mesons.

$$V_{ij}(r) = V_\pi(r) + V_\sigma(r) + V_\omega(r) + V_\rho(r). \tag{21}$$

The Dirac representation [32, 35] for functions of mesons will be used in addition to $V_\omega(r) = V_\rho(r)$, and get the following representations.

$$V_{ps}(r) = \gamma_i^o \gamma_i^5 \gamma_j^o \gamma_j^5 J_{ps}, \tag{22}$$

$$V_\sigma(r) = -\gamma_i^o \gamma_j^o J_\sigma, \tag{23}$$

$$V_\omega(r) = \gamma_i^o \gamma_j^o \vec{\gamma}_i^{\,\mu} \vec{\gamma}_j^{\,\mu} J_\omega, \tag{24}$$

$$\vec{\gamma}_i^{\,\mu} \vec{\gamma}_j^{\,\mu} = \left[\gamma_i^o \gamma_j^o - \vec{\gamma}_i \vec{\gamma}_j\right], \tag{25}$$

where

$$\begin{aligned}
\beta \equiv \gamma_i^o &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \vec{\gamma}_i = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \gamma^5 \\
&= i\gamma^o \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.
\end{aligned} \tag{26}$$

Substitute Equations (22) and (26) into Equation (21) to get the expectation value by three potentials V_π, V_σ, and V_ω.

$$\begin{aligned}
&\left\langle F_\alpha F_\gamma \middle| V_{ij}(r) \middle| \widetilde{F_\beta F_\delta} \right\rangle \\
&\quad = \left\langle F_\alpha F_\gamma \middle| V_{ps} \middle| \widetilde{F_\beta F_\delta} \right\rangle + \left\langle F_\alpha F_\gamma \middle| V_s \middle| \widetilde{F_\beta F_\delta} \right\rangle \\
&\quad + \left\langle F_\alpha F_\gamma \middle| 2V_v \middle| \widetilde{F_\beta F_\delta} \right\rangle = \left\langle F_\alpha F_\gamma \middle| V_\pi \middle| \widetilde{F_\beta F_\delta} \right\rangle \\
&\quad + \left\langle F_\alpha F_\gamma \middle| V_\sigma \middle| \widetilde{F_\beta F_\delta} \right\rangle + \left\langle F_\alpha F_\gamma \middle| 2V_\omega \middle| \widetilde{F_\beta F_\delta} \right\rangle, \\
&\left\langle F_\alpha F_\gamma \middle| V_{ij}(r) \middle| \widetilde{F_\beta F_\delta} \right\rangle \\
&= \langle (\phi_\alpha \quad \chi_\alpha) | \langle (\phi_\gamma \quad \chi_\gamma) | \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_i \\
&\quad \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_j \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_j J_\pi \left| \begin{pmatrix} \phi_\beta \\ \chi_\beta \end{pmatrix} \right\rangle \left| \begin{pmatrix} \phi_\delta \\ \chi_\delta \end{pmatrix} \right\rangle \\
&\quad + \langle (\phi_\alpha \quad \chi_\alpha) | \langle (\phi_\gamma \quad \chi_\gamma) | \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}_i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_j \\
&\quad \cdot J_\sigma \left| \begin{pmatrix} \phi_\beta \\ \chi_\beta \end{pmatrix} \right\rangle \left| \begin{pmatrix} \phi_\delta \\ \chi_\delta \end{pmatrix} \right\rangle + 2\langle (\phi_\alpha \quad \chi_\alpha) | \langle (\phi_\gamma \quad \chi_\gamma) | \\
&\quad \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_i \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_j J_\omega \left| \begin{pmatrix} \phi_\beta \\ \chi_\beta \end{pmatrix} \right\rangle \left| \begin{pmatrix} \phi_\delta \\ \chi_\delta \end{pmatrix} \right\rangle \\
&\quad - 2\langle (\phi_\alpha \quad \chi_\alpha) | \langle (\phi_\gamma \quad \chi_\gamma) | \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}_i \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}_j \\
&\quad \cdot J_\omega \left| \begin{pmatrix} \chi_\beta \\ -\phi_\beta \end{pmatrix} \right\rangle \left| \begin{pmatrix} \phi_\delta \\ \chi_\delta \end{pmatrix} \right\rangle.
\end{aligned} \tag{27}$$

According to the relation between ϕ and χ in Equation (14), one obtains

$$\begin{aligned}
&\left\langle F_\alpha F_\gamma \middle| V_{ij}(r) \middle| \widetilde{F_\beta F_\delta} \right\rangle \\
&\quad = \left\langle \phi_\alpha \phi_\gamma \middle| \frac{1}{4m^2c^2} \left[J_\pi (\vec{\sigma}_j.\vec{p}_j) - (\vec{\sigma}_j.\vec{p}_j) J_\pi (\vec{\sigma}_i.\vec{p}_i) \right.\right. \\
&\quad - (\vec{\sigma}_i.\vec{p}_i) J_\pi (\vec{\sigma}_j.\vec{p}_j) + (\vec{\sigma}_i.\vec{p}_i)(\vec{\sigma}_j.\vec{p}_j) J_\pi \Big] \Big| \\
&\quad - J_\sigma + 2J_\omega + \frac{1}{4m^2c^2} \Big[(\vec{\sigma}_i.\vec{p}_i) J_\sigma (\vec{\sigma}_i.\vec{p}_i) \\
&\quad + (\vec{\sigma}_j.\vec{p}_j) J_\sigma (\vec{\sigma}_j.\vec{p}_j) + 2(\vec{\sigma}_i.\vec{p}_i) J_\omega (\vec{\sigma}_i.\vec{p}_i) \\
&\quad + (\vec{\sigma}_j.\vec{p}_j) J_\omega (\vec{\sigma}_j.\vec{p}_j) - 2J_\omega (\vec{\sigma}_i.\vec{\sigma}_j)(\vec{\sigma}_j.\vec{p}_j) \\
&\quad \cdot (\vec{\sigma}_i.\vec{p}_i) - 2(\vec{\sigma}_j.\vec{p}_j) J_\omega (\vec{\sigma}_i.\vec{\sigma}_j)(\vec{\sigma}_i.\vec{p}_i) \\
&\quad - 2(\vec{\sigma}_i.\vec{p}_i) J_\omega (\vec{\sigma}_i.\vec{\sigma}_j)(\vec{\sigma}_j.\vec{p}_j) - 2(\vec{\sigma}_i.\vec{p}_i) \\
&\quad \left.\left. \cdot (\vec{\sigma}_j.\vec{p}_j) J_\omega (\vec{\sigma}_i.\vec{\sigma}_j) \right] \middle| \widetilde{\phi_\beta \phi_\delta} \right\rangle.
\end{aligned} \tag{28}$$

Defining the momentum for each nucleon (i, j) $\vec{p}_i = \vec{p}_r + 1/2\vec{p}_R$, $\vec{p}_j = -\vec{p}_r + 1/2\vec{p}_R$ [27, 36]. Substituting those relations into Equation (28), where $\vec{p}_r = \vec{p}$ and $(\vec{\sigma}_i.\vec{p}_R)(\vec{\sigma}_i.\vec{p}_R) = p_R^2$, we obtain

$$\begin{aligned} V_{ij}(r) = &-J_\sigma + 2J_\omega + \frac{1}{4m^2c^2}\Big[\left(\vec{\sigma}_i.\vec{p}\right)J_\sigma\left(\vec{\sigma}_i.\vec{p}\right) \\ &+ \left(\vec{\sigma}_j.\vec{p}\right)J_\sigma\left(\vec{\sigma}_j.\vec{p}\right) + 2\left(\vec{\sigma}_i.\vec{p}\right)J_\omega\left(\vec{\sigma}_i.\vec{p}\right) \\ &+ 2\left(\vec{\sigma}_j.\vec{p}\right)J_\omega\left(\vec{\sigma}_j.\vec{p}\right) + 2J_\omega\left(\vec{\sigma}_i.\vec{\sigma}_j\right) \\ &\cdot\left(\vec{\sigma}_j.\vec{p}\right)\left(\vec{\sigma}_i.\vec{p}\right) + 2\left(\vec{\sigma}_i.\vec{p}\right)J_\omega\left(\vec{\sigma}_i.\vec{\sigma}_j\right) \\ &\cdot\left(\vec{\sigma}_j.\vec{p}\right) + 2\left(\vec{\sigma}_j.\vec{p}\right)J_\omega\left(\vec{\sigma}_i.\vec{\sigma}_j\right)\left(\vec{\sigma}_i.\vec{p}\right) \\ &+ 2\left(\vec{\sigma}_i.\vec{p}\right)\left(\vec{\sigma}_j.\vec{p}\right)J_\omega\left(\vec{\sigma}_i.\vec{\sigma}_j\right) + 2J_\omega\left(\vec{\sigma}_i.\vec{\sigma}_j\right) \\ &\cdot\left(\vec{\sigma}_j.\vec{p}\right)\left(\vec{\sigma}_i.\vec{p}_R\right) - 2J_\omega\left(\vec{\sigma}_i.\vec{\sigma}_j\right)\left(\vec{\sigma}_j.\vec{p}_R\right) \\ &\cdot\left(\vec{\sigma}_i.\vec{p}\right) + 2\left(\vec{\sigma}_j.\vec{p}\right)\left(\vec{\sigma}_i.\vec{p}_R\right)J_\omega\left(\vec{\sigma}_i.\vec{\sigma}_j\right) \\ &- 2\left(\vec{\sigma}_i.\vec{p}\right)\left(\vec{\sigma}_j.\vec{p}_R\right)J_\omega\left(\vec{\sigma}_i.\vec{\sigma}_j\right)\Big] + \frac{1}{8m^2c^2} \\ &\cdot\Big[\left(\vec{\sigma}_i.\vec{p}\right)\left(\vec{\sigma}_i.\vec{p}_R\right)J_\sigma + J_\sigma\left(\vec{\sigma}_i.\vec{p}_R\right)\left(\vec{\sigma}_i.\vec{p}\right) \\ &- \left(\vec{\sigma}_j.\vec{p}\right)\left(\vec{\sigma}_j.\vec{p}_R\right)J_\sigma - J_\sigma\left(\vec{\sigma}_j.\vec{p}_R\right)\left(\vec{\sigma}_j.\vec{p}\right) \\ &+ 2\left(\vec{\sigma}_i.\vec{p}\right)\left(\vec{\sigma}_i.\vec{p}_R\right)J_\omega + 2J_\omega\left(\vec{\sigma}_i.\vec{p}_R\right)\left(\vec{\sigma}_i.\vec{p}\right) \\ &- 2\left(\vec{\sigma}_j.\vec{p}\right)\left(\vec{\sigma}_j.\vec{p}_R\right)J_\omega - 2J_\omega\left(\vec{\sigma}_j.\vec{p}_R\right)\left(\vec{\sigma}_j.\vec{p}\right) \\ &+ \vec{p}_R^{\,2}J_\sigma + 2\vec{p}_R^{\,2}J_\omega - 2J_\omega\left(\vec{\sigma}_i.\vec{\sigma}_j\right)\left(\vec{\sigma}_j.\vec{p}_R\right)\left(\vec{\sigma}_i.\vec{p}_R\right)\Big] \\ &+ \frac{1}{4m^2c^2}\Big[-J_\pi\left(\vec{\sigma}_j.\vec{p}\right)\left(\vec{\sigma}_i.\vec{p}\right) + \left(\vec{\sigma}_j.\vec{p}\right)J_\pi\left(\vec{\sigma}_i.\vec{p}\right) \\ &+ \left(\vec{\sigma}_i.\vec{p}\right)J_\pi\left(\vec{\sigma}_j.\vec{p}\right) - \left(\vec{\sigma}_i.\vec{p}\right)\left(\vec{\sigma}_j.\vec{p}\right)J_\pi\Big]. \end{aligned} \tag{29}$$

We will apply some important relations [37]

$$\begin{aligned} \left(\vec{\sigma}_1.\vec{A}\right)\left(\vec{\sigma}_1.\vec{B}\right) &= A.B + i\vec{\sigma}_1(A\times B), \\ \left(\vec{\sigma}_1.\vec{A}\right)^2 &= A^2, \\ \left(\vec{\sigma}_1.\vec{A}\right)\left(\vec{\sigma}_2.\vec{A}\right) &= \frac{2}{\hbar^2}(S.A)^2 - A^2, \end{aligned} \tag{30}$$

$$\begin{aligned} &\left(\vec{\sigma}.\vec{A}\right)F(r)\left(\vec{\sigma}.\vec{A}\right) \\ &\quad = F(r)A^2 - i\hbar\left\{\nabla F(r).A + i\vec{\sigma}[(\nabla F(r))\times A]\right\}. \end{aligned}$$

Include these relations in potential equation. We substitute every term by using the relation of angular momentum $\vec{L} = \vec{r}\times\vec{p}$, $\vec{\sigma} = 2\vec{S}/\hbar$ (where $\vec{S}$ is the total spin operator), $\vec{p} = -i\hbar\nabla$, and $\nabla J_\sigma = 1/r(dJ_\sigma/dr)r$. According to the previous relations, where $\vec{\sigma}_j^{\,2} = \vec{\sigma}_x^{\,2} + \vec{\sigma}_y^{\,2} + \vec{\sigma}_z^{\,2} = 1$ as triplet case for two nucleons,

$$\begin{aligned} &\left(\vec{\sigma}_j.\vec{p}\right)J_\omega(r)\left(\vec{\sigma}_j\vec{\sigma}_j\right)\left(\vec{\sigma}_j.\vec{p}\right) \\ &\quad = \left(\vec{\sigma}_j.\vec{p}\right)J_\omega(r)\vec{\sigma}_j^{\,2}\left(\vec{\sigma}_i.\vec{p}\right) \\ &\quad = -3J_\omega(r)p^2 + 3\hbar^2\left\{\frac{dJ_\omega}{dr}\frac{d}{dr}\right\} - \frac{6}{r}\frac{dJ_\omega}{dr}\left[\vec{S}_j.\vec{L}\right], \\ &\left(\vec{\sigma}_i.\vec{p}\right)J_\omega(r)\left(\vec{\sigma}_i\vec{\sigma}_j\right)\left(\vec{\sigma}_j.\vec{p}\right) \\ &\quad = -3J_\omega(r)p^2 + 3\hbar^2\left\{\frac{dJ_\omega}{dr}\frac{d}{dr}\right\} - \frac{6}{r}\frac{dJ_\omega}{dr}\left[\vec{S}_i.\vec{L}\right]. \end{aligned} \tag{31}$$

With total spin operator $\vec{S}$ and the meson function $J(r)$, using [38] $(\vec{S}.\vec{p})^2 = (\vec{S}.\hat{n})^2p^2$, $(\vec{\sigma}_i.\vec{\sigma}_j) = 2/\hbar^2(S^2-3)$ and $\vec{S}.\vec{L} = \hbar^2/2[J(J+1)-L(L+1)-S(S+1)]$. Quantum mechanics have a magnificent tool; this tool is the harmonic oscillator which is capable of being solved in closed form, and it has generally useful approximations and exact solutions of different problems [39]. It solves the differential equations in quantum mechanics. We have the energy of harmonic oscillator $(\hbar\omega(2n+l+3/2))$ which equals the kinetic energy $(p^2/2m)$ added to the potential energy $((1/2)m\omega^2x^2)$ to simplify the solution and get the result. It is slitted in relative harmonic oscillator energy $\hbar\omega(2n+l+3/2) = p^2/2\mu + 1/2\mu\omega^2r^2$ [30, 40], with ω that is the angular frequency and center of mass contribution in harmonic oscillator energy $\hbar\omega(2N+L+3/2) = p^2/2M + 1/2M\omega^2R^2$. We suppose the nucleons have average masses $m_n + m_p/2$, so the relative mass $\mu = m_1m_2/m_1+m_2 = m/2$, and center mass $M = m_1 + m_2 = 2m$.

$$\begin{aligned} V_{ij}(r) = &-J_\sigma + 2J_\omega + \frac{1}{8\mu^2c^2}\Bigg[-\hbar^2\left\{\frac{dJ_\sigma}{dr}\frac{d}{dr}\right\} \\ &+ \frac{1}{r}\frac{dJ_\sigma}{dr}\left[\frac{\hbar^2}{2}[J(J+1)-L(L+1)-S(S+1)]\right] \\ &+ 4\hbar^2\left\{\frac{dJ_\omega}{dr}\frac{d}{dr}\right\} - 2\frac{2}{r}\frac{dJ_\omega}{dr}\Bigg[\frac{\hbar^2}{2}[J(J+1)-L(L+1) \\ &- S(S+1)]\Bigg]\Bigg] + \frac{1}{4\mu c^2}\Bigg[J_\sigma(r)\left(\frac{\vec{p}^{\,2}}{2\mu}\right) - 2J_\omega(r)\left(\frac{\vec{p}^{\,2}}{2\mu}\right) \\ &+ 4J_\omega\left(2\vec{S}\left(\vec{S}+1\right)-3\right)\left(\frac{2}{\hbar^2}\left(\vec{S}.\hat{n}\right)^2-1\right)\left(\frac{\vec{p}^{\,2}}{2\mu}\right) \\ &+ 4\left(\frac{2}{\hbar^2}\left(\vec{S}.\hat{n}\right)^2-1\right)\left(\frac{\vec{p}^{\,2}}{2\mu}\right)J_\omega\left(2\vec{S}\left(\vec{S}+1\right)-3\right)\Bigg] \\ &+ 2\frac{1}{Mc^2}\Bigg[-2\left(2\vec{S}\left(\vec{S}+1\right)-3\right)J_\omega(r)\left(\frac{2}{\hbar^2}\left(\vec{S}.\hat{n}\right)^2 \\ &-1\right)\left(\frac{\vec{p}_R^{\,2}}{2M}\right) + \left(\frac{p_R^2}{2M}\right)J_\sigma + \left(\frac{p_R^2}{2M}\right)J_\omega\Bigg] ++\frac{1}{4m^2c^2} \\ &\cdot\Bigg[-J_\pi\left(2\left(\vec{S}.\hat{n}\right)^2p^2 + J_\pi p^2 - 2\hbar^2\left(2\vec{S}\left(\vec{S}+1\right)-3\right) \\ &\cdot\frac{dJ_\pi}{dr}\frac{d}{dr} - 2\left(\vec{S}.\hat{n}\right)^2p^2J_\pi + p^2J_\pi\Bigg]. \end{aligned} \tag{32}$$

The wave functions of the two nucleons of Equation (28) should be treated as follows:

$$\begin{aligned}\left\langle \phi_\alpha(r_i)\phi_\gamma(r_j)\right| &= \sum_{m_{l_\alpha} m_{s_\alpha}} \sum_{m_{l_\gamma} m_{s_\gamma}} \left(l_\alpha s_\alpha m_{l_\alpha} m_{s_\alpha} \mid j_\alpha M_\alpha\right)\left(l_\gamma s_\gamma m_{l_\gamma} m_{s_\gamma} \mid j_\gamma M_\gamma\right) \\ &\cdot \left\langle \phi_{n_\alpha l_\alpha m_{l_\alpha}}(r_i)\phi_{n_\gamma l_\gamma m_{l_\gamma}}(r_j)\right|\left\langle \chi^{1/2}_{m_{s_\alpha}}\chi^{1/2}_{m_{s_\gamma}}\right|\left\langle \widehat{P}_{T_\alpha}\widehat{P}_{T_\gamma}\right|. \end{aligned} \tag{33}$$

See Appendix B to have the final formula.

$$\begin{aligned}\left\langle \phi_\alpha(r_i)\phi_\gamma(r_j)\right| &= \sum_{m_{l_\alpha} m_{s_\alpha}} \sum_{m_{l_\gamma} m_{s_\gamma}} \sum_{JM}\sum_{\lambda\mu}\sum_{nlNL}\sum_{mM}\sum_{sm_s}\sum_{T} \left(l_\alpha s_\alpha m_{l_\alpha} m_{s_\alpha} \mid j_\alpha M_\alpha\right) \\ &\cdot \left(l_\gamma s_\gamma m_{l_\gamma} m_{s_\gamma} \mid j_\gamma M_\gamma\right)\left(l_\alpha l_\gamma m_{l_\alpha} m_{l_\gamma} \mid \lambda\mu\right) \\ &\cdot \langle n_\alpha l_\alpha n_\gamma l_\gamma \mid NLnl\rangle (lSm_l m_S \mid JM)(LlMm \mid \lambda\mu) \\ &\cdot \left(s_\alpha s_\gamma m_{s_\alpha} m_{s_\gamma} \mid SM_s\right)\left(\chi_\alpha\chi_\gamma T_\alpha T_\gamma \mid TM_T\right) \\ &\cdot \langle \phi_{NLM}(R)\phi_{nlm}(r)| \left\langle \chi^{1/2}_{m_{s_\alpha}}(i,j)\right|\left\langle \widehat{P}_T(i,j)\right|. \end{aligned} \tag{34}$$

The bracket $\langle n_\alpha l_\alpha n_\gamma l_\gamma \mid NLnl\rangle$ represents the Talmi-Moshinsky bracket. The same treatment for the ket part $|\phi_\beta(r_i)\phi_\delta(r_j)\rangle$ to have

$$\begin{aligned}\left|\phi_\beta(r_i)\phi_\delta(r_j)\right\rangle &= \sum_{m_{l_\beta} m_{s_\beta}} \sum_{m_{l_\delta} m_{s_\delta}} \sum_{JM}\sum_{\lambda\mu}\sum_{nlNL}\sum_{mM}\sum_{sm_s}\sum_{T} \left(l_\beta s_\beta m_{l_\beta} m_{s_\beta} \mid j_\beta M_\beta\right) \\ &\cdot \left(l_\delta s_\delta m_{l_\delta} m_{s_\delta} \mid j_\delta M_\delta\right)\left(l_\beta l_\delta m_{l_\beta} m_{l_\delta} \mid \lambda\mu\right) \\ &\cdot \langle n_\beta l_\beta n_\delta l_\delta \mid NLnl\rangle (lSm_l m_S \mid JM)(LlMm \mid \lambda\mu) \\ &\cdot \left(s_\beta s_\delta m_{s_\beta} m_{s_\delta} \mid SM_s\right)\left(\chi_\beta\chi_\delta T_\beta T_\delta \mid TM_T\right) \\ &\cdot |\phi_{NLM}(R)\phi_{nlm}(r)\rangle \left|\chi^{1/2}_{m_{s_\beta}}(i,j)\right|\left.\widehat{P}_T(i,j)\right\rangle. \end{aligned} \tag{35}$$

We have the wave function $\phi_{NLM}(R) = R_{NLM}(R)Y_{NLM}(\vartheta,\varphi)$ as radial part (R) and angular part (Y) for center of mass coordinates, the wave function $\phi_{nlm}(r) = R_{nlm}(r)Y_{nlm}(\vartheta,\varphi)$ as radial part (R), and angular part (Y) for relative coordinates. For the two-nucleon interaction formula through the exchange of four mesons where $\overrightarrow{p}_{ij} = \overrightarrow{p}$ and A is the mass number of the required nuclei, we define the bracket $\langle \chi^s_{m_s}(i,j) \mid \chi^s_{m_s}(i,j)\rangle = 1$, $\langle \widehat{P}_T(i,j) \mid \widehat{P}_T(i,j)\rangle = 1$ and $\langle Y_{NLM}Y_{nlm} \mid Y_{NLM}Y_{nlm}\rangle = 1$ as the terms of equation depend on (r).

We have the formula of radial wave function which involves the length parameter $b = \sqrt{\hbar/m\omega}$ with angular frequency ω. and the associated Laguerre polynomial $L_n^{l+1/2}$.

$$\begin{aligned} R_{nl} &= \left[\frac{2n!}{\Gamma(n+l+3/2)}\right]^{1/2}\left(\frac{1}{b}\right)^{3/2}\left(\frac{r}{b}\right)^{l} \\ &\cdot \exp\left(\frac{-1}{2}\left(\frac{r}{b}\right)^2\right)L_n^{l+1/2}\left(\frac{r}{b}\right)^2. \end{aligned} \tag{36}$$

The differentiation of Radial function equals

$$\frac{d}{dr}R_{nl}(r) = \frac{l}{r}R_{n,l} - \frac{r}{b^2}R_{n,l} - \frac{2r\sqrt{n'}}{b^2}R_{n-1,l+1}. \tag{37}$$

Define the operator $\overrightarrow{S}.\widehat{n}$ [38] as

$$\begin{aligned}(S.\widehat{n})Y^{LS}_{JM}(\vartheta,\varphi) &= \frac{-\hbar}{2}\left[\left(\frac{(J+L+S+2)(J+L+S+1)(J-L+S)(-J+L+S+1)}{(2L+1)(2L+3)}\right)^{1/2} Y^{L+S}_{JM}(\vartheta,\varphi)\right. \\ &\left. + \left(\frac{(J+L+S+1)(J+L-S)(J-L+S+1)(-J+L+S)}{(2L-1)(2L+1)}\right)^{1/2} Y^{L-S}_{JM}(\vartheta,\varphi)\right]. \end{aligned} \tag{38}$$

The meson degrees of freedom have some static functions J_k for description, but here we choose GY and SPED for meson k and $(k = \pi, \sigma, \omega, \rho)$.

$$(J_k)_{\mathrm{GY}} = g_k\hbar c\left(\frac{\exp(-\mu_k r)}{r} - \frac{\exp(-\lambda_k r)}{r}\left(1 + \frac{\lambda^2 - \mu_k^2}{2\lambda_k}r\right)\right), \tag{39}$$

where the meson-nucleon coupling constant g_k^2, the cut-off λ_k, and the mass of the meson are associated with $\mu_k = mc/\hbar$. The second function has the form [24]

$$(J_k)_{\mathrm{SPED}} = g_k\hbar\left(\frac{\lambda_k^2}{\lambda_k^2 - \mu_k^2}\right)\left(\frac{\exp(-\mu_k r)}{r} - \frac{\exp(-\lambda_k r)}{r}\right). \tag{40}$$

5. Theoretical Analysis of CDP

There is an explicit spin dependence for the instanton interaction unlike one-gluon exchange, and it contains a projector on spin $S = 0$ states. The distribution of this interaction represented with $\delta(\overrightarrow{r})$ replaced by a Gaussian function with range Λ.

$$\delta(r) = \frac{1}{\Lambda^3}\frac{1}{\pi^{3/2}}\exp\left(\frac{-r^2}{\Lambda^2}\right), \tag{41}$$

where Λ is the range of the pairing force (QCD scale parameter).

$$V_I(r) = 8\begin{pmatrix} g & \sqrt{2}g' \\ \sqrt{2}g' & 0 \end{pmatrix}\delta(r), \tag{42}$$

where g and g' are two dimensioned coupling constants according to quark flavors. This equation is under condition of $(l = s = 0, I = 0)$, where l, s, and I denote angular momentum, spin, and isotopic spin quantum numbers, respectively, for $n\bar{n}$ pair, and the form of instanton contributions represents as [41]. The pairing force depends on the value of the parameters g and g', if we set g for strange flavor with symbol s and g' for nonstrange flavor with symbol n. The Hamiltonian contains diagonal parts in the isoscalar space $(|n\bar{n}\rangle, |s\bar{s}\rangle)$.

$$H = \begin{pmatrix} \sum_{i=1}^{3} \sqrt{\vec{p}_i^{\,2} + m_i^2} & 0 \\ 0 & \sum_{i=1}^{3} \sqrt{\vec{p}_i^{\,2} + m_i^2} \end{pmatrix} + \frac{1}{2}\left[\frac{-k}{r} + ar + C\right]\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + V_I\left(\vec{r}\right), \quad (43)$$

having the coupling constant as

$$g' = \frac{3}{8} g_{\text{eff}}(n). \quad (44)$$

The parameter g_{eff} denotes the strength and is defined as [16]

$$g_{\text{eff}} = \left(\frac{4}{3}\pi^2\right)^2 \int_0^{\rho_c} d\rho d_o(\rho)\rho^2 \times \left(m_i^o - \rho^2 c_i\right), \quad (45)$$

where $d_o(\rho)$ is a function instanton density of the instanton size ρ. For three colors and three flavors, this quantity is given in [16], m_i^o is the current mass of flavor i, and the quark condensate for this flavor is $c_i = (2/3)\pi^2\langle\bar{q}_i q_i\rangle$, $\langle\bar{q}_i q_i\rangle$ (nonvanishing expectation values). The integration over ρ_c which is the maximum size of the instanton is

$$d_o(\rho) = \left(3.63 \times 10^3\right)\left(\frac{8\pi^2}{g^2(\rho)}\right)^6 \exp\left(\frac{8\pi^2}{g^2(\rho)}\right), \quad (46)$$

where

$$\left(\frac{8\pi^2}{g^2(\rho)}\right) = 9 \ln\left(\frac{1}{\Lambda\rho}\right) - \frac{32}{9} \ln\left(\ln\left(\frac{1}{\Lambda\rho}\right)\right). \quad (47)$$

The constituent masses are the renormalization of quarks' masses which demonstrate the contribution of the constituent masses [16].

$$m_n = m_n^o + \Delta m_n + \delta_n. \quad (48)$$

m_n^o is the current mass of nonstrange quark, and Δm_n is the contribution of constituent mass [13] with free parameter δ_n added to the running masses. The contribution of the constituent masses has the following formula.

$$\Delta m_n = \frac{3}{4}\pi^2 \int_0^{\rho_c} d\rho d_o(\rho)\rho^2 \left(m_n^o - \rho^2 c_n\right)\left(m_s^o - \rho^2 c_s\right). \quad (49)$$

It is important to replace the dimensional instanton size [14] as $x = \Lambda\rho$ with a dimensionless quantity with using the definition of $d_o(\rho)$.

$$\alpha_n(x_c) = \int_0^{x_c} dx \left[9 \ln\left(\frac{1}{x}\right) - \frac{32}{9} \ln\left(\ln\left(\frac{1}{x}\right)\right)\right]^6 x^n \left[\ln\left(\frac{1}{x}\right)\right]^{-32/9}. \quad (50)$$

This dimensionless integration should still have small value of lnln-term. It is involved in the parameters g' and Δm_n.

$$g' = \frac{\delta\pi^2}{2\Lambda^3}\left[m_n^o \alpha_{11}(x_c) - \frac{c_n}{\Lambda^2}\alpha_{13}(x_c)\right],$$
$$\Delta m_n = \frac{\delta}{\Lambda}\left[m_n^o m_s^o \alpha_9(x_c) - \frac{c_n m_s^o + c_s m_n^o}{\Lambda^2}\alpha_{11}(x_c) + \frac{c_n c_s}{\Lambda^4}\alpha_{13}(x_c)\right]. \quad (51)$$

In the functions $\alpha_9(x_c)$, $\alpha_{11}(x_c)$, $\alpha_{13}(x_c)$ given in [14], the m_s^o is the constituent mass of strange flavor and also the c_s is the quark condensate related to the strange flavor. It is supposed that the quark as an effective degrees of freedom is dressed by the gluon and quark-antiquark pair clouds (constituent masses), and it is natural to express the probability density of quark configuration as a Gaussian function around its average position.

$$\rho_i(r) = \frac{1}{\left(\gamma_i\sqrt{\pi}\right)^{3/2}} \exp\left(\frac{-r^2}{\gamma_i^2}\right), \quad (52)$$

where $\rho_i(r)$ is the probability density not the instanton size as previous with γ_i the size parameter, and it is dependent on the quark mass flavor (n for nonstrange flavor and s for strange flavor). The operator for the quark in positions r_1 and r_2 is replaced by effective one after double convolution of the bare operator with the density functions ρ_i and ρ_j. This can be performed by using the dressed expression $\tilde{O}_{ij}(r)$ of the bare operator $O_{ij}(r)$ which depends only on the relative distance $r_{ij} = r_i - r_j$ between quarks [13].

$$\tilde{O}_{ij} = \int dr O_{ij}\left(r'\right)\rho_{ij}\left(r_{ij} - r'\right). \quad (53)$$

The convolution procedure supposed to remain the center of mass fixed during it and that the ρ_{ij} tends to a delta function at the limit of an infinitely large γ_{ij} [42].

$$\tilde{\delta}(r) = \frac{1}{\left(\gamma_{ij}\sqrt{\pi}\right)^3} \exp\left(\frac{-r^2}{\gamma_{ij}^2}\right). \quad (54)$$

This formula resembles the previous form of the probability density of Gaussian form, but with parameter γ_{ij}. The convolution (a function derived from two given functions by integration that expresses how the shape of one is modified by the other) of two Gaussian functions with size parameter γ_i and γ_j is also a Gaussian function. After convolution with the quark density, the Cornell-dressed potential has the following form:

$$\tilde{V}_C(r) = -k\frac{\operatorname{erf}\left(r/\gamma_{ij}\right)}{r} + ar\left[\frac{\gamma_{ij}\exp\left(-r^2/\gamma_{ij}^2\right)}{\sqrt{\pi}r} + \left(1+\frac{\gamma_{ij}^2}{2r^2}\right)\operatorname{erf}\left(\frac{r}{\gamma_{ij}}\right)\right] + C, \quad (55)$$

with the error function erf and $\gamma_{ij} = \sqrt{\gamma_i^2 + \gamma_j^2}$, where $\gamma_i = 1/(\gamma_i\sqrt{\pi})^{3/2}\exp(-r^2/\gamma_i^2)$ and $\gamma_j = 1/(\gamma_i\sqrt{\pi})^{3/2}\exp(-r^2/\gamma_i^2)$.

6. Results and Discussion

Table 1 represents the group of parameters used for $(\pi, \sigma, \omega, \rho)$ mesons. The sets of parameters are I and II that include mass of meson, the coupling constant (g), and the cut-off parameter (λ).

The parameters listed in Table 2 is related to the quark-quark potential through the CDP which is added to the nucleon-nucleon potential in the hybrid model.

The two-body force is a simple model to reveal the hidden physics of the atomic systems and also the nuclear systems. Our work boils down to simple fact of constructing more realistic model that contains all possible degrees of freedom in some light nuclei such as deuteron $^2H_1$ and helium $^4H_2$. So, we include the interaction between two baryons which is bounded in a hadron, and each baryon contains three bounded quarks. The nucleon-nucleon interaction is well introduced by the exchange of mesons with the OBE model. At long range of this interaction, it is supposed to be due to the exchange of pion-meson (pseudoscalar meson) followed by the effect of scalar meson (σ) in attractive attitude at the medium range. The attractive behavior has to face an opposite behavior to maintain the stability of nuclei, so the short range of this interaction is affected by a repulsive behavior due to the exchange of vector mesons such as (ω, ρ) and QCD effects. The potential is elaborated to calculate the ground-state energies for the ^{2}H and ^{4}He nuclei. We have examined the OBEP to calculate the ground-state energy of ^{2}H and ^{4}He nuclei using two static meson functions (GY and SPED) with two sets of parameters listed in Table 1 which shows the different sets of the used parameters and for different exchange mesons, (σ, ω) mesons, (π, σ, ω) mesons, and $(\pi, \sigma, \omega, \rho)$.We have mentioned that there is an effect of QCD at the short range of nucleon-nucleon interaction via three bounded quarks interacted between each other. The so-called Funnel potential or Cornell potential is simple and the best model for the description of Charmonium system, but in our hybrid model without coupling between mesons and quarks, it gives too high values and the effect of our model is a destructive one. When we tried to apply the idea of hybrid model with the aid of the instanton-induced interaction, it really gives us a transition probability for the interaction of quark-quark interaction in small scale comparing with the confinement scale. It is indeed similar to the tunneling effect with possibility of treating the instanton interaction as a field configuration between quarks and antiquarks in the ground states. This interaction is also applied on the light quarks not only the quark-antiquark. So, it is useful for us in our model as the proton or neutron is a hadron of three light quarks.

The instanton interaction is included in the CDP, giving us a small value ranged between −0.15 MeV in the case of ^{2}H and −0.25 MeV in the case of ^{4}He around the boundaries of the hadron. Our results are shown in Tables 3 and 4 with the effect of CDP with parameters of Table 2 besides the exchange of mesons through OBEP. Generally, the effect is encouraged, and it improves the ratios for the ground-state energies of the deuteron and helium nuclei in all cases with different parameters of meson degrees of freedom and different functions GY and SPED.

We have determined the ratio (Rat) to ensure the accuracy between the calculated results and the experimental data.

$$\text{Rat} = \frac{E_{\text{theor}}}{E_{\text{exp.}}}, \quad (56)$$

Table 1: The meson parameters for OBEP for different sets [43].

	Ref	Meson	Mass MeV	Coupling constant $g_i/4\pi$	Cut off parameter λ MeV
Set I	[43]	π	138.03	14.9	2000
		σ	700	16.07	2000
		ω	782.6	28	1300
		ρ	769	1.7	1100
Set II	[43]	π	138.03	14.40	1700
		σ	710	18.37	2000
		ω	782.6	24.50	1850
		ρ	769	0.9	1850

Table 2: The quark parameters for the instanton-induced interaction with the CDP [14].

Parameters	Unit	Values in baryon
a	GeV^2	0.16803
K		0.79801
C	GeV	-0.96701
m_m	GeV	0.378
y_n	$GeV-^1$	0.68101

Table 3: The ground-state energy of deuteron with the hybrid model related to quark and meson degrees of freedom.

Parameter sets [43]	Meson exchange	Hybrid (GY+CDP)	Hybrid (SPED+CDP)	Exp. [44–46]	Ratio GY+CDP	Ratio SPED+CDP	$B.E/A$ GY+CD	$B.E/A$ SPED+CDP
I	(σ,ω)	-3.066	-2.191		1.3785	0.985	1.533	1.096
II		-2.496	-2.123		1.122	0.955	1.248	1.0615
I	(π,σ,ω)	-2.349	-2.398	-2.224	1.056	1.078	1.175	1.199
II		-2.318	-2.354		1.042	1.058	1.159	1.177
I	(π,σ,ω,ρ)	-2.277	-2.317		1.024	1.042	1.138	1.158
II		-2.027	-2.529		0.911	1.1371	1.033	1.264

Table 4: The ground-state energy of helium with the hybrid model.

Parameter sets [43]	Meson exchange	Hybrid (GY+CDP)	Hybrid (SPED+CDP)	Exp. [47]	Ratio GY+CDP	Ratio SPED+CDP	$B.E/A$ GY+CDP	$B.E/A$ SPED+CDP
I	(σ,ω)	-22.622	-20.488		1.1089	1.0043	5.655	5.122
II		-23.001	-21.806		1.1275	1.0689	5.750	5.541
I	(π,σ,ω)	-22.887	-20.625	-20.4 ± 0.3	1.1219	1.0110	5.7217	5.156
II		-22.121	-20.587		1.084	1.0091	5.530	5.147
I	(π,σ,ω,ρ)	-19.905	-19.9888		0.9757	0.9798	4.976	4.997
II		-19.6244	-21.0417		0.9619	1.03145	4.906	5.260

Table 5: The ground-state energy of ^{2}H nucleus based on OBEP.

Parameter sets [43]	Meson	Present work (GY)	Present work (SPED)	Others	Exp. [44–46]	Ratio GY	Ratio SPED	$B.E/A$ GY	$B.E/A$ SPED
I	(σ,ω)	-2.916	-2.041	-2.215		1.311	0.918	1.458	1.0205
II		-3.486	-1.973	[49]	-2.224	1.567	0.887	1.743	0.987
I	(π,σ,ω)	-2.199	-2.248			0.989	1.011	1.099	1.124
II		-2.168	-2.204			0.975	0.991	1.084	1.102
I	(π,σ,ω,ρ)	-2.127	-2.167			0.9563	0.974	1.063	1.084
II		-1.877	-2.379			0.8438	1.069	0.938	1.189

Table 6: The ground-state energy of ^{4}He nucleus through OBE.

Parameter sets [43]	Meson	Present work (GY)	Present work (SPED)	Others [50]	Exp. [47]	Ratio GY	Ratio SPED	E/A GY	E/A SPED
I	(σ,ω)	-22.372	-20.238			1.0966	0.992	5.593	5.0595
II		-22.751	-21.556	-21.385	-20.4 ± 0.3	1.115	1.057	5.6877	5.389
I	(π,σ,ω)	-22.637	-20.375			1.109	0.999	5.659	5.0937
II		-21.871	-20.337			1.072	0.997	5.4677	5.08425
I	(π,σ,ω,ρ)	-19.655	-19.7388			0.9497	0.9675	4.9137	4.9347
II		-19.3744	-20.7917			0.9497	1.0192	4.8436	5.1979

where E_{theor} is the calculated ground state and E_{exp} the experimental one. We can also determine the binding energy per nucleon $B.E/A$ for the studied nuclei as [48]

$$\frac{B.E}{A} = -\frac{E_{g.s.}}{A}, \tag{57}$$

where the mass number is A and the total ground state energy is $E_{g.s.}$. The results of the OBEP are listed in Tables 5 and 6 in comparison with other theoretical and experimental data. The ratio between the present work and experimental one is estimated for both cases, in other words by using the potential extracted from GY and SPED functions.

At first, the OBEP depended on the cancelation of σ meson and ω meson, and the results are satisfied for the ground-state energies of ^{2}H nucleus as in Table 5 with two different sets of meson s' parameters; but here, we also tried to include the OBEP through the exchange of three mesons (π,σ,ω) and four mesons (π,σ,ω,ρ), and the results are listed in Table 5 with two static functions for the meson: GY and SPED functions. The preferable value of deuteron ground state is in the case of using three mesons by using SPED function for parameter I. It is noticed that the case of exchange three mesons gives closer value than the case of exchange of four mesons, demonstrating the effect of π meson as an attractive one to be clear than the effect of ρ meson. This behavior is reasonable for light nuclei.

The ^{4}He nucleus has the same manner as the ^{2}H nucleus with preferable values ranged from 20.1 to 20.7, and that is listed in Table 6.

The ratio is getting a better result for going on more massive nuclei and encouraged for our potential. The calculation of binding energy per nucleon serves our idea of being the OBEP with three and four mesons in the case of SPED function and gives satisfied values for deuteron and helium nuclei comparing with the experimental one as it is for deuteron $B.E/A = 1.112$ and for helium is $B.E/A \simeq 5.1$.

Tables 3 and 4 have the effect of adding quark degrees of freedom to the meson degrees of freedom in a hybrid model for all previous cases, The values are reasonable and the best value of the hybrid model with the exchange of four mesons in the case of parameter II for ^{2}H nucleus when we apply the GY function than others. The results are different for ^{4}He nucleus, and we have the value of SPED function with the exchange of two and three mesons in the hybrid model with the parameter I to be the preferred one. It is obvious from Tables 3 and 4 the ground energy is close to the data in the case of SPED function for set I and set II in comparison with the experimental data. The ^{4}He nucleus has little different manner, and the theoretical values of the hybrid model are more cleared than in ^{2}H nucleus. It is noticed that the quark-quark interaction improves the values with GY function. We concluded that the used model is well-defined and compatible with the data and even than other models (see [51, 52]).

7. Conclusion

In the framework of quasi-relativistic formulation, the meson exchange potential helps in obtaining a potential with few number of parameters to calculate the ground state for the light nuclei deuteron and helium using two (σ, ω), three (π, σ, ω), and four $(\pi, \sigma, \omega, \rho)$ meson exchanges. In addition, it was shown that a self-consistent treatment of the semirelativistic nucleon wave function in nuclear state has a great importance in calculations. The difference in masses of σ and ω mesons would not seriously change the main aspect of the concept of relativistic or semirelativistic interaction, providing an average potential of cancelation of the repulsive meson (ω) and the attractive meson (σ) in conjunction with a weak long-range effect (π). The work with OBEP in Dirac-Hartree-Fock equation gives a close relationship to other recent approaches, based upon different formalisms which tended to support this direction. The nuclear properties are being clear in our trail to include more two mesons to describe the NN interaction through our potential. The SPED function has a good ability to give us the better shapes of our potential and also better values for energies. We hope that our potential represents a base for NN interaction with different ranges of energies in the following search. The ground-state energies for ^{2}H and ^{4}He nuclei are successfully determined through this work and give us a hope to continue with more massive nuclei. The QCD model is based on one-gluon exchange process besides the interaction of instanton that supplemented the confinement. The Cornell-dressed potential represents the interaction between quarks through the exchange of pseudoscalar mesons (instantons) under controlling of one-gluon exchange process. Our semirelativistic hybrid model is encouraged for light nuclei, and the instanton-induced interaction caused to construct a link of quark-quark interaction to the nucleon-nucleon interaction for small scale around the hadron boundaries. The effect of adding the QQ interaction on the ground-state energies is ranged from 6.7 for ^{2}H nucleus to 1.2 for ^{4}He nucleus; this is a small effect, and it is expected to be vanished for massive nuclei.

Appendix

A. Kinetic Energy

We deal with the kinetic energy as a relative kinetic energy T_{ij} which is related to the relative momentum $p_{ij} = 1/2(p_1 - p_2)$ with the momentum of the first nucleon p_1 and momentum of the second nucleon p_2, and the center-of-mass momentum $p_R = p_1 + p$. Therefore, the relative kinetic energy has the formula

$$\begin{aligned}
T_{ij} = T_i - T_{c.m} &= \sum_i \frac{p_i^2}{2m} - \frac{(\sum_i p_i)2}{2mA} \\
&= \sum_i \frac{p_i^2}{2m} - \frac{1}{2mA}\left[\sum_i p_i^2 + \sum_{i<j} 2p_i p_j\right] \\
&= \sum_i \frac{p_i^2}{2m} - \frac{1}{2mA}\left[\sum_i p_i^2 + \sum_{i<j}\left(p_i^2 + p_j^2 - 4p_{ij}^2\right)\right] \\
&= \sum_i \frac{p_i^2}{2m} - \frac{1}{2mA}\left[\sum_i p_i^2 + (A-1)\sum_i p_i^2 - 4\sum_{i<j} p_{ij}^2\right] \\
&= \sum_i \frac{p_i^2}{2m} - \frac{1}{2mA}\left[A\sum_i p_i^2 - 4\sum_{i<j} p_{ij}^2\right] = \frac{2}{mA}\sum_{i<j} p_{ij}^2,
\end{aligned} \tag{A.1}$$

where T_i is the kinetic energy of particles in the system, $T_{c.m}$ is the kinetic energy of the center-of-mass effect, m is the mass of the nucleus, and A is the mass number of nucleus.

B. Wave Function with the Clebsch-Gordan Coefficient

The wave functions for two nucleons i and j have a form with Clebsch-Gordan coefficients.

$$\begin{aligned}
\left\langle \phi_\alpha(r_i)\phi_\gamma(r_j)\right| = \sum_{m_{l_\alpha} m_{s_\alpha}} \sum_{m_{l_\gamma} m_{s_\gamma}} & \left(l_\alpha s_\alpha m_{l_\alpha} m_{s_\alpha} \mid j_\alpha M_\alpha\right) \\
& \cdot \left(l_\gamma s_\gamma m_{l_\gamma} m_{s_\gamma} \mid j_\gamma M_\gamma\right) \\
& \cdot \left\langle \phi_{n_\alpha l_\alpha m_{l_\alpha}}(r_i)\phi_{n_\gamma l_\gamma m_{l_\gamma}}(r_j)\right| \\
& \cdot \left\langle \chi^{1/2}_{m_{s_\alpha}} \chi^{1/2}_{m_{s_\gamma}}\right|\left\langle \widehat{P}_{T_\alpha}\widehat{P}_{T_\gamma}\right|,
\end{aligned} \tag{B.1}$$

where (l) is the orbital angular momentum, s_γ is the spin, the total angular momentum $j_\alpha = l_\alpha + s_\alpha$, $j_\gamma = l_\gamma + s_\gamma$, and $M_\alpha = m_{l_\alpha} + m_{s_\alpha}$ in which m_{l_α} is the projection of orbital quantum number, m_{s_α} is the projection of spin quantum number, $M_\gamma = m_{l_\gamma} + m_{s_\gamma}$, and $\widehat{P}_{T_\alpha}$ is the function of isotopic spin. The two wave functions are not connected and depend on r_i, r_j, so the two wave functions need to be connected.

$$\left\langle \phi_\alpha(r_i)\phi_\gamma(r_j)\right| = \sum_{m_{l_\alpha} m_{s_\alpha}} \sum_{m_{l_\gamma} m_{s_\gamma}} \sum_{\lambda\mu} \left(l_\alpha s_\alpha m_{l_\alpha} m_{s_\alpha} \mid j_\alpha M_\alpha\right) \cdot \left(l_\gamma s_\gamma m_{l_\gamma} m_{s_\gamma} \mid j_\gamma M_\gamma\right)\left(l_\alpha l_\gamma m_{l_\alpha} m_{l_\gamma} \mid \lambda\mu\right) \cdot \left\langle \phi_{n_\alpha l_\alpha m_{l_\alpha}}(r_i)\phi_{n_\gamma l_\gamma m_{l_\gamma}}(r_j)\right|\left\langle \chi^{1/2}_{m_{s_\alpha}} \chi^{1/2}_{m_{s_\gamma}}\right| \cdot \left\langle \widehat{P}_{T_\alpha}\widehat{P}_{T_\gamma}\right|. \tag{B.2}$$

With $\lambda = l_\alpha + l_\gamma$ and $\mu = m_{l_\alpha} + m_{l_\gamma}$, we can change the special coordinates for each wave functions to become one wave that depends on relative mass and center of mass.

$$\left\langle \phi_\alpha(r_i)\phi_\gamma(r_j)\right| = \sum_{m_{l_\alpha} m_{s_\alpha}} \sum_{m_{l_\gamma} m_{s_\gamma}} \sum_{\lambda\mu} \sum_{nlNL} \left(l_\alpha s_\alpha m_{l_\alpha} m_{s_\alpha} \mid j_\alpha M_\alpha\right) \cdot \left(l_\gamma s_\gamma m_{l_\gamma} m_{s_\gamma} \mid j_\gamma M_\gamma\right)\left(l_\alpha l_\gamma m_{l_\alpha} m_{l_\gamma} \mid \lambda\mu\right) \cdot \left\langle n_\alpha l_\alpha n_\gamma l_\gamma \mid NLnl\right\rangle\left(s_\alpha s_\gamma m_{s_\alpha} m_{s_\gamma} \mid SM_s\right) \cdot \left\langle \phi_{NLnl}(r,R)\right|\left\langle \chi^{1/2}_{m_{s_\alpha}} \chi^{1/2}_{m_{s_\gamma}}\right|\left\langle \widehat{P}_{T_\alpha}\widehat{P}_{T_\gamma}\right|, \tag{B.3}$$

where $\langle n_\alpha l_\alpha n_\gamma l_\gamma \mid NLnl\rangle$ is the Talmi-Moshinsky bracket, NL is total center of mass, and nl is total relative. The wave function $\phi_{NLnl}(r,R)$ can be spitted in to the form

$$\left\langle \phi_\alpha(r_i)\phi_\gamma(r_j)\right| = \sum_{m_{l_\alpha} m_{s_\alpha}} \sum_{m_{l_\gamma} m_{s_\gamma}} \sum_{JM} \sum_{\lambda\mu} \sum_{nlNL} \sum_{mM} \left(l_\alpha s_\alpha m_{l_\alpha} m_{s_\alpha} \mid j_\alpha M_\alpha\right) \cdot \left(l_\gamma s_\gamma m_{l_\gamma} m_{s_\gamma} \mid j_\gamma M_\gamma\right)\left(l_\alpha l_\gamma m_{l_\alpha} m_{l_\gamma} \mid \lambda\mu\right) \cdot \left\langle n_\alpha l_\alpha n_\gamma l_\gamma \mid NLnl\right\rangle\left(s_\alpha s_\gamma m_{s_\alpha} m_{s_\gamma} \mid SM_s\right) \cdot \left(lSm_l m_S \mid JM\right)\left(LlMm \mid \lambda\mu\right) \cdot \left\langle \phi_{NLM}(R)\phi_{nlm}(r)\right|\left\langle \chi^{1/2}_{m_{s_\alpha}} \chi^{1/2}_{m_{s_\gamma}}\right|\left\langle \widehat{P}_{T_\alpha}\widehat{P}_{T_\gamma}\right|. \tag{B.4}$$

As L gives the total orbital quantum number in center of mass, l gives the total orbital quantum number in relative coordinates and $S = s_i + s_j$ is the total spin. Relative to the spin functions and isospin functions to be connected, we have to use them as follows:

$$\left\langle \phi_\alpha(r_i)\phi_\gamma(r_j)\right| = \sum_{m_{l_\alpha} m_{s_\alpha}} \sum_{m_{l_\gamma} m_{s_\gamma}} \sum_{JM} \sum_{\lambda\mu} \sum_{nlNL} \sum_{mM} \sum_{sm_s} \sum_{T} \cdot \left(l_\alpha s_\alpha m_{l_\alpha} m_{s_\alpha} \mid j_\alpha M_\alpha\right)\left(l_\gamma s_\gamma m_{l_\gamma} m_{s_\gamma} \mid j_\gamma M_\gamma\right) \cdot \left(l_\alpha l_\gamma m_{l_\alpha} m_{l_\gamma} \mid \lambda\mu\right)\left\langle n_\alpha l_\alpha n_\gamma l_\gamma \mid NLnl\right\rangle \cdot \left(lSm_l m_S \mid JM\right)\left(LlMm \mid \lambda\mu\right) \cdot \left(s_\alpha s_\gamma m_{s_\alpha} m_{s_\gamma} \mid SM_s\right)\left(\chi_\alpha \chi_\gamma T_\alpha T_\gamma \mid TM_T\right) \cdot \left\langle \phi_{NLM}(R)\phi_{nlm}(r)\left\langle \chi^{1/2}_{m_{s_\alpha}}(i,j)\right|\left\langle \widehat{P}_T(i,j)\right|\right., \tag{B.5}$$

with the isotopic spin $T_{\text{proton}} = -1/2$ and $T_{\text{neutron}} = 1/2$.

Data Availability

The experimental data used to support the calculations of this study are included within the article and are taken from published articles.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] K. Hanna, S. H. M. Sewailem, R. Hussien, L. I. Abou-Salem, and A. G. Shalaby, "The ground-state calculations for some nuclei by mesonic potential of nucleon- nucleon interaction," *Advances in High Energy Physics*, vol. 2020, Article ID 3271975, 15 pages, 2020.

[2] R. Machleidt, K. Holinde, and C. Elester, "The Bonn meson-exchange model for the nucleon–nucleon interaction," *Physics Reports*, vol. 149, no. 1, pp. 1–89, 1987.

[3] R. de Tourreil, B. Rouben, and D. W. L. Sprung, "Super-soft-core nucleon-nucleon interaction with *π*-, *ρ*\- and *gw*-exchange contributions," *Nuclear Physics A*, vol. 242, no. 3, pp. 445–460, 1975.

[4] M. M. Nagels, T. A. Rijken, and J. D. de Swart, "Low-energy nucleon-nucleon potential from Regge-pole theory," *Physical Review D*, vol. 17, no. 3, pp. 768–776, 1978.

[5] K. Erkelenz, "Current status of the relativistic two-nucleon one boson exchange potential," *Physics Reports*, vol. 13, no. 5, pp. 191–258, 1974.

[6] K. Holinde and R. Machleidt, "Momentum-space OBEP, two-nucleon and nuclear matter data," *Nuclear Physics A*, vol. 247, no. 3, pp. 495–520, 1975.

[7] J. Fleischer and J. A. Tjon, "Bethe-Salpeter equation for elastic nucleon-nucleon scattering," *Physical Review D*, vol. 21, no. 1, pp. 87–94, 1980.

[8] M. J. Zuilhof and J. A. Tjon, "Electromagnetic properties of the deuteron and the Bethe-Salpeter equation with one-boson exchange," *Physics Review C*, vol. 22, no. 6, pp. 2369–2382, 1980.

[9] W. Lucha, F. F. Schoberl, and D. Gromes, "Bound states of quarks," *Physics Reports*, vol. 200, no. 4, pp. 127–240, 1991.

[10] G. Plante and A. F. Antippa, "Analytic solution of the Schrödinger equation for the Coulomb-plus-linear potential. I. The wave functions," *Journal of Mathematical Physics*, vol. 46, no. 6, article 062108, 2005.

[11] D. Kang and E. Won, "Precise numerical solutions of potential problems using the Crank-Nicolson method," *Journal of Computational Physics*, vol. 227, no. 5, pp. 2970–2976, 2008.

[12] J. Su, Y. Dong, and S. Wu, "Test of a general Lorentz structure of the qq-bar confinement in calculations of meson spectra," *Journal of Physics G*, vol. 18, no. 8, pp. 1347–1354, 1992.

[13] F. Brau and C. Semay, "Light meson spectra and instanton-induced forces," *Physical Review D*, vol. 58, no. 3, article 034015, 1998.

[14] C. Semay, F. Brau, and B. Silvestre-Brac, "Baryon spectra with instanton induced forces," *Physical Review C*, vol. 64, no. 5, article 055202, 2001.

[15] W. H. Blask, U. Bohn, M. G. Huber, B. C. Metsch, and H. R. Petry, "Hadron spectroscopy with instanton induced quark forces," *Zeitschrift für Physik A Atomic Nuclei*, vol. 337, no. 3, pp. 327–335, 1990.

[16] C. R. Munz, J. Resag, B. C. Metsch, and H. R. Petry, "A Bethe-Salpeter model for light mesons: spectra and decays," *Nuclear Physics A*, vol. 578, no. 3-4, pp. 418–440, 1994.

[17] L. P. Fulcher, "Matrix representation of the nonlocal kinetic energy operator, the spinless Salpeter equation and the Cornell potential," *Physical Review D*, vol. 50, no. 1, pp. 447–453, 1994.

[18] G. T. Hooft, "Computation of the quantum effects due to a four-dimensional pseudoparticle," *Physical Review D*, vol. 14, no. 12, pp. 3432–3450, 1976.

[19] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, "Instanton density in a theory with massless quarks," *Nuclear Physics B*, vol. 163, pp. 46–56, 1980.

[20] T. Grafke, R. Grauer, and T. Schafer, "The instanton method and its numerical implementation in fluid mechanics," *Journal of Physics A: Mathematical and Theoretical*, vol. 48, no. 33, article 333001, 2015.

[21] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, "Microscopic theory of superconductivity," *Physics Review*, vol. 106, no. 1, pp. 162–164, 1957.

[22] T. Schafer and E. V. Shuryak, "Instantons in QCD," *Reviews of Modern Physics*, vol. 70, pp. 324–425, 1998.

[23] L. D. Miller and A. E. S. Green, "Relativistic self-consistent meson field theory of spherical nuclei," *Physics Review C*, vol. 5, no. 1, pp. 241–252, 1972.

[24] M. Jaminan, C. Mahaux, and P. Rochus, "Optical-model potential in a relativistic quantum field model," *Physics Review C*, vol. 22, no. 5, pp. 2027–2042, 1980.

[25] J. M. Arias and M. Lozano, *An Advanced Course in Modern Nuclear Physics*, Springer-Verlag, Berlin Heidelberg, 2001.

[26] U.-G. Meibner, "Modern theory of nuclear forces," *Nuclear Physics A*, vol. 751, p. 149, 2005.

[27] T. Neff, H. Feldmeier, and W. Horiuchi, "Short-range correlations in nuclei with similarity renormalization group transformations," *Physical Review C*, vol. 92, no. 2, article 024003, 2015.

[28] T. Myo, H. Toki, and K. Ikeda, "Tensor-optimized shell model with bare nucleon-nucleon interaction for ^{4}He," *Progress of Theoretical Physics*, vol. 121, no. 3, pp. 511–531, 2009.

[29] J. Zhang, *Arizona State University*, p. 14, 2014.

[30] S. Gartenhaus and C. Schwaetz, "Center-of-mass motion in many-particle systems," *Physical Review*, vol. 108, p. 2, 1957.

[31] A. Bouyssy, J. F. Mathiot, and N. V. Giai, "Relativistic description of nuclear systems in the Hartree-Fock approximation," *Physical Review C*, vol. 36, no. 1, pp. 380–401, 1987.

[32] W. H. Long, P. Ring, N. V. Giai, and J. Meng, "Relativistic Hartree-Fock-Bogoliubov theory with density dependent meson-nucleon couplings," *Physics Review C*, vol. 81, no. 2, article 024308, 2010.

[33] A. E. S. Green and T. Sawada, "Meson theoretic N–N interactions for nuclear physics," *Reviews of Modern Physics*, vol. 39, no. 3, pp. 594–610, 1967.

[34] S. B. Khadkikar and V. B. Kample, "Centre of mass motion and single particle separation energies in Hartree-Fock theory," *Nuclear Physics A*, vol. 225, no. 2, pp. 352–356, 1974.

[35] A. Anselm and N. Dombey, "Fermionic bound states and pseudoscalar exchange," *Journal of Physics G: Nuclear and Particle Physics*, vol. 25, no. 3, pp. 513–523, 1999.

[36] F. Gross, J. W. V. Orden, and K. Holinde, "Relativistic one-boson-exchange model for the nucleon-nucleon interaction," *Physics Review C*, vol. 45, no. 5, pp. 2094–2132, 1992.

[37] J. Raynal, 2004, https://arxiv.org/abs/nucl-th/0407060.

[38] D. A. Varshalovich, A. N. Moskalev, and V. K. Khersonski, *Quantum Theory of Angular Momentum*, World Scientific Co. Pte. Ltd., 1988.

[39] M. W. Kirson, "Oscillator parameters in nuclei," *Nuclear Physics A*, vol. 781, no. 3-4, pp. 350–362, 2007.

[40] K. Gad, "Ground-state properties of closed-shell nucleus 56Ni with realistic nucleon–nucleon interactions," *Annals of Physics*, vol. 327, no. 10, pp. 2403–2410, 2012.

[41] C. Semay and B. Silvestre-Brac, "Mesons in the relativistic flux tube model," *Physics Review D*, vol. 52, no. 11, pp. 6553–6561, 1995.

[42] C. Semay and B. Silvestre-Brac, "Potential models and meson spectra," *Nuclear Physics A*, vol. 618, no. 4, pp. 455–482, 1997.

[43] J. W. Negele and E. Vogt, *Advances in Nuclear Physics*, Plenum Press, New York-London, 1989.

[44] M. Garçon and J. W. Van Orden, "The Deuteron: Structure and Form Factors," in *Advances in Nuclear Physics. Advances in the Physics of Particles and Nuclei*, J. W. Negele and E. W. Vogt, Eds., vol. 26, Springer, Boston, MA, 2001.

[45] G. L. Greene, E. G. Kessler Jr., R. D. Deslattes, and H. Brner, "New determination of the deuteron binding energy and the neutron mass," *Physical Review Letters*, vol. 56, p. 819, 1986.

[46] E. G. Kessler, M. S. Deweya, R. D. Deslattesa et al., "The deuteron binding energy and the neutron mass," *Physics Letters A*, vol. 255, no. 4-6, pp. 221–229, 1999.

[47] H. Tyren, S. Kullander, O. Sundberg, R. Ramachandran, P. Isacsson, and T. Berggren, "Quasi-free proton-proton scattering in light nuclei at 460 MeV," *Nuclear Physics*, vol. 79, no. 2, pp. 321–373, 1966.

[48] K. Gad, "A study of the ground-state energy of 4He with nucleon⍰nucleon potentials.," *Annalen der Physik*, vol. 326, p. 2474, 2011.

[49] F. Huang and W. L. Wang, "Nucleon-nucleon interaction in a chiral SU(3) quark model revisited," *Physical Review D*, vol. 98, no. 7, article 074018, 2018.

[50] K. Kowalski, D. J. Dean, M. H. Jensen, T. Papenbrock, and P. Piecuch, "Coupled cluster calculations of ground and excited states of nuclei," *Physical Review Letters*, vol. 92, p. 13, 2004.

[51] R. B. Wiringa, "Variational calculations of few-body nuclei," *Physics Review C*, vol. 43, no. 4, pp. 1585–1598, 1991.

[52] I. Alhagaish, A. Abu-Nada, F. Afaneh, and M. Hasan, "Ground state properties of ^{4}He and ^{12}C nuclei at equilibrium and at large static compression at zero temperature using Nijmegen and Reid Soft Core nucleon-nucleon interactions," https://arxiv.org/abs/1803.04650v1.